$$\varepsilon \leqslant \min\{\varepsilon_{\parallel}, \varepsilon_{\perp}\}, \quad \varepsilon_{\perp} = \frac{1}{N}.$$

NOTATION

a, λ, $c\rho$, effective thermal diffusivity, thermal conductivity, and volumetric heat capacity of a quasihomogeneous body; a_i, λ_i, thermal diffusivity and thermal conductivity of the i-th component; a_m, value obtained by measuring thermal diffusivity of the inhomogeneous specimen; δa, relative methodical error of the measurements; l, thickness of a flat specimen; N, number of plates in the representative element of system A; h, characteristic microdimension of system B; $\varepsilon_{\parallel} = h/l$; $\nu = \lambda_2/\lambda_1$; $\beta = a_2/a_1$.

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GENERALIZATION OF DATA ON THE DEPENDENCE OF THE COORDINATION NUMBER ON THE POROSITY IN FILLINGS OF SINTERED OR PRESSED GRANULAR MATERIALS

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An approximation equation is proposed for estimating the mean number of contacts with a particle in granular systems.

The dependence of the "coordination number" N_c (the average number of contacts with a particle) on the porosity P in free fillings of sintered or pressed granular materials has been given considerable attention in publications on general problems of physics and geometry [1-3], powder metallurgy [4-6], chemical technology [7, 8], the theory of heat and mass transfer [9-14], and the properties of alloys [15, 16].

For an analytical estimate of the value of the coordination number semiempirical and purely empirical approximation relations are employed [1, 4, 5, 8], which agree quite well with experimental data, as a rule, only over a narrow range of variation of the porosity $0.3 \leqslant P \leqslant 0.5$ (Fig. 1).

The use of existing relationships [1, 4, 5, 8] to estimate the value of the coordination number for other values of the porosity leads to considerable disagreements with the results of measurements, or, in general, leads to absurd results (zero or negative values of the coordination numbers). The latter is obviously due to the fact that in the publications mentioned the suitability of the theoretical relations employed was verified by comparison with experimental data over a small range of variation of the porosity, limited by the framework of the problems considered.

A direct geometrical solution of the problem of determining the coordination number which agrees well with experiment over a quite wide range of variation of porosity $0.26 < P < 1.0$ (see curve 21 in Fig. 1) was proposed in [10]. If we analyze existing measurement results, we find a quite clear relationship between the change in coordination number N_c and the porosity.

Practically all the experimental points (29 out of 32) can be generalized quite well by the following approximating relationship (see curve 22 in Fig. 1):

$$N_c = [10 \sin^3 0.5\pi (1 - P) + 3] \pm 1, \quad 0 \leqslant P < 1, \tag{1}$$

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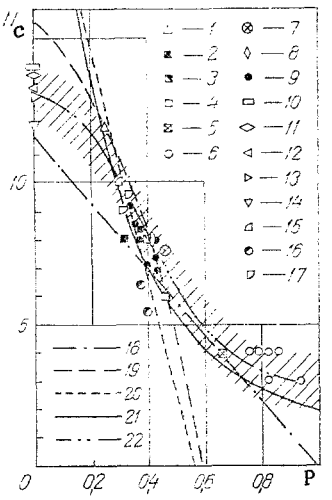

Fig. 1. Coordination number N_c as a function of the porosity P. Experiment: 1-5) values of N_c for ordered stacking of spheres (hpc or fcc N_c = 12, vss N_c = 8, orthorhombic N_c = 8, simple cubic N_c = 6, diamond N_c = 4, respectively); 6-17) random structures: 6) [2], 7) [5], 8) [2], 9) [3, 6, 8, 11, 12], 10-14) [3], 15-17) [2, 3]. Theoretical: 18) [9], 19) [5], 20) [8], 21) [10], 22) from Eq. (1); the hatched region represents the limits of the region of spread of $N_c(P)$.

where the first term on the right side describes the mean value of $N_c(P)$. The spread from the mean value does not exceed ±1 over practically the whole range of variation of porosity. The lower values of coordination numbers N_c = 5.5 for P = 0.4, N_c = 6.4 for P = 0.37, and N_c = 8 for P = 0.32 relate to structures obtained in a particular way (loose packing, when rolling spheres into one) [8]. It can be seen that the proposed approximation formula (1) is suitable for granular structures both with ordered (points 1-5 in Fig. 1) and random (points 6-17 in Fig. 1) packing of the grains.

A numerical solution of the problem of determining the dependence of the coordination number on the porosity in unordered fillings of spheres of one size was obtained in [13]. Due to the complexity of the calculations, specific values of the coordination numbers were obtained for two values of the porosity (Fig. 1).

The fluctuations in the value of the coordination number are symmetrical with respect to the most probable values (N_c = 7 for P = 0.4, and N_c = 8 for P = 0.36) and constitute ±1.5 units, i.e., about 20% for a confidence coefficient of 0.67. The latter agrees well with estimates using Eq. (1).

Note that for highly porous granular systems (P > 0.7) the mean value of the coordination number tends to small values: as $P \to 1$, $N_c \to 2$ [10], N_c = 4 [1], and N_c = 3 [2]. The value of N_c = 2 is in effect a limiting minimum value for which the "chain" structure, representing the continuous set of contacting particles, should retain mechanical stability (the particles are not "suspended" in space).

In fact, as was shown in [14], a structure of highly porous fillings (Fig. 2a) is formed by a "carcas" with a relatively dense packing (0.4 < P < 0.6) of randomly arranged but contacting particles and with coarse cavities, localized within the thickness of the carcas or penetrating it.

In the region P > 0.7 the small values of the coordination numbers $2 < N_c < 4$ are rather the consequence of the averaging of N_c over a volume of the system with a large fraction of coarse cavities, while the true values of the coordination numbers lie in the range $4 < N_c < 5$. The latter agrees with the minimum values of the coordination numbers in unordered alloys of salts [15] and metals [16].

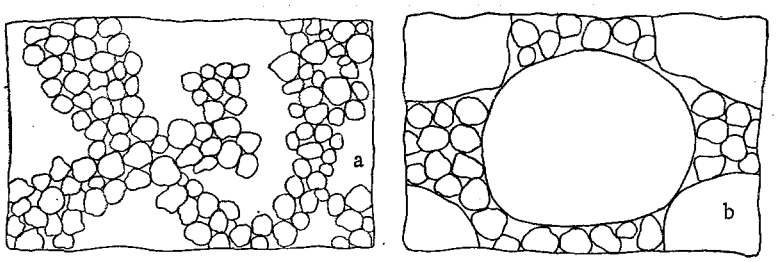

Fig. 2. Structure of highly porous (a) $P > 0.7$ and polydisperse granular systems of low porosity (b) $P < 0.3$ in a state of free filling or slightly sintered.

The possibility of using relation (1) to estimate the value of N_c for polydisperse systems with $d_{max}/d_{min} \geqslant 5$ (Fig. 2b) is of interest. In this case the coordination number for coarse particles may increase considerably (many fine particles are in contact with a large particle), whereas for fine particles it will approach minimum values. The result of the effect of opposite tendencies to change $N_c(P, d_{max}/d_{min})$ for large and small particles can probably be studied theoretically, although in the opinion of Nayak and Tien [13], this is an extremely complex problem. To a first approximation N_c can be estimating using Eq. (1).

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